What is a Catalanimal?

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ICERM: What is...? Seminar

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Resources

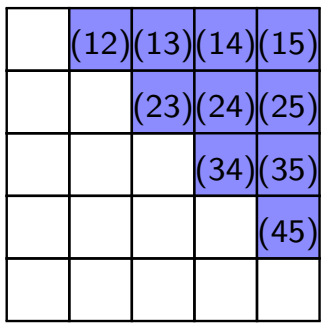
- My IPAC 2024 Lectures:
  `https://www2.math.upenn.edu/~jhaglund/IPAC/2024.html`
- Anna Pun's 2021 Lectures:
  `https://www2.math.upenn.edu/~jhaglund/IPAC/2021.html`

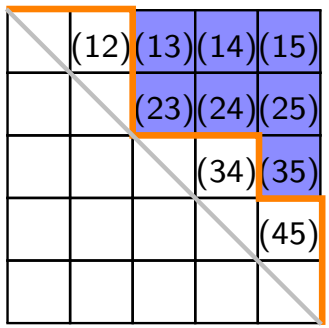
Hall-Littlewood Examples

$$
\begin{aligned}
H_{(4,2)} &= \mathrm{pol}\left(\boldsymbol{\sigma}\left(\frac{x_1^4 x_2^2}{1-tx_1/x_2}\right)\right) \\
&= \mathrm{pol}\left(\boldsymbol{\sigma}\left(x_1^4x_2^2 + tx_1^5x_2 + t^2x_1^6x_2^0 + t^3x_1^7x_2^{-1} + \cdots\right)\right) \\
&= \mathrm{pol}\left(\chi_{42} + t\chi_{51} + t^2\chi_{60} + t^3\chi_{7,\text{-}1} + \cdots\right) \\
&= s_{42} + ts_{51} + t^2 s_6\,.
\end{aligned}
$$

$$
\begin{aligned}
H_{(4,2,1)} &= \mathrm{pol}\left(\boldsymbol{\sigma}\left(\frac{x_1^4 x_2^2 x_3^1}{(1-tx_1/x_2)(1-tx_1/x_3)(1-tx_2/x_3)}\right)\right) \\
&\cdots \\
&= s_{421} + ts_{43} + ts_{511} + (t^2+t)s_{52} + (t^3+t^2)s_{61} + t^4 s_7
\end{aligned}
$$

Root Ideals

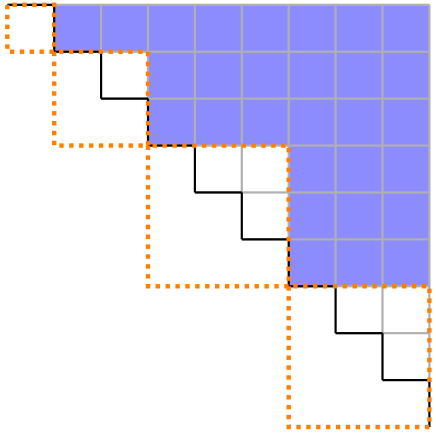




Catalan Function Examples

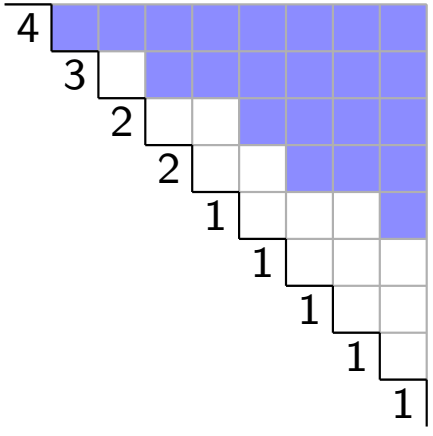
- Parabolic Hall-Littlewood polynomials $H_{\eta,\lambda} = H(R_\eta^+, \lambda)$

$\eta = (1, 2, 3, 3)$ $\qquad R_\eta^+ =$



- k-Schur functions $s_\lambda^{(k)} = H(\Delta^k(\lambda), \lambda)$

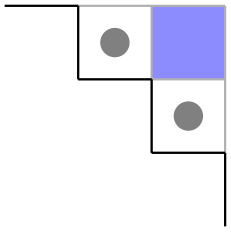
$\Delta^4(432211111) =$



Catalanimal Example

Special case: $R_+ = R_q = R_t \supseteq R_{qt}$

- $R_t \setminus R_{qt}$
- R_{qt}



$$H(R_+, R_+, \{\alpha_{13}\}, 111)$$

$$= \operatorname{pol} \boldsymbol{\sigma} \left(\frac{x_1 x_2 x_3 (1 - qt x_1/x_3)}{\prod_{1 \le i < j \le 3} (1 - q x_i/x_j)(1 - t x_i/x_j)} \right)$$

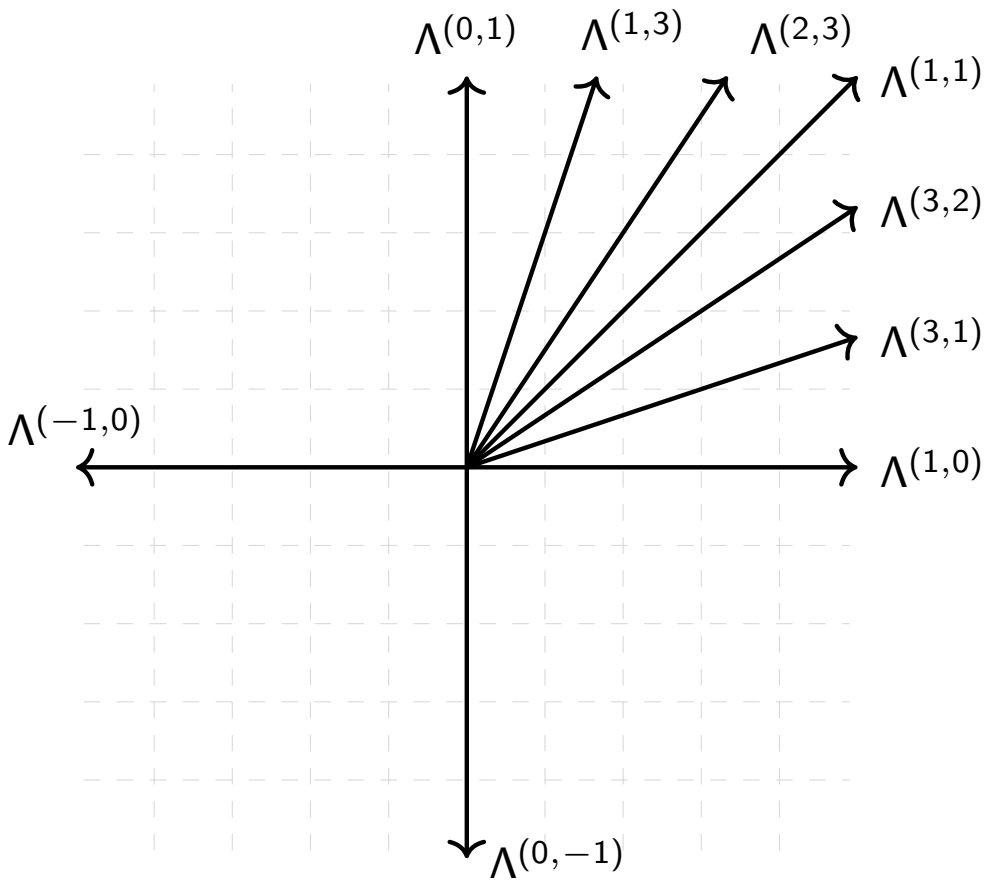
$$= \cdots$$

$$= s_{111} + (q^2 + qt + t^2 + q + t) s_{21} + (q^3 + q^2 t + qt^2 + t^3 + qt) s_3$$

$$= \omega \nabla e_3$$

Elliptic Hall Algebra $\mathcal{E}$

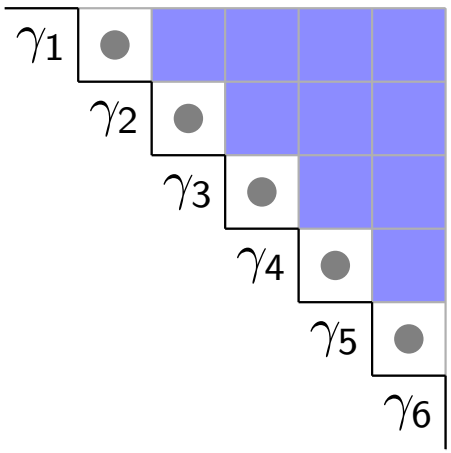
$\Lambda^{(m,n)} = \Lambda(X^{m,n}) \cong \Lambda$



Negut Catalanimal

Special case: $R_+ = R_q = R_t \supseteq R_{qt}$

- $R_t \setminus R_{qt}$
- R_{qt}



$$H(R_+, R_+, [R_+, R_+], \gamma) = \mathrm{pol}\, \boldsymbol{\sigma} \left(\frac{x^\gamma \prod_{i+1<j} (1 - qtx_i/x_j)}{\prod_{i<j} (1 - qx_i/x_j)(q - tx_i/x_j)} \right)$$

Some $(1,1)$-Cuddly Catalanimals

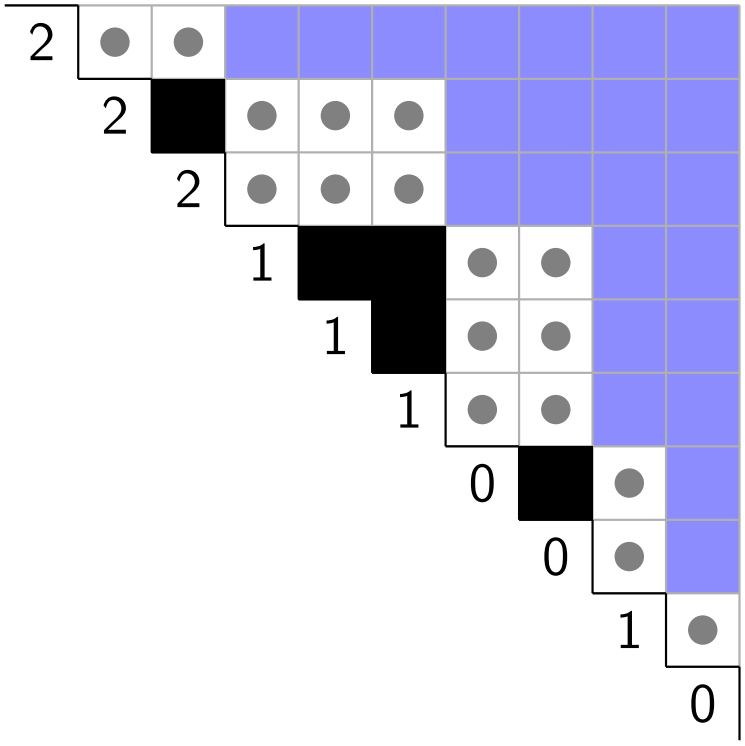
Special case: $R_+ \supseteq R_q \supseteq R_t \supseteq R_{qt}$

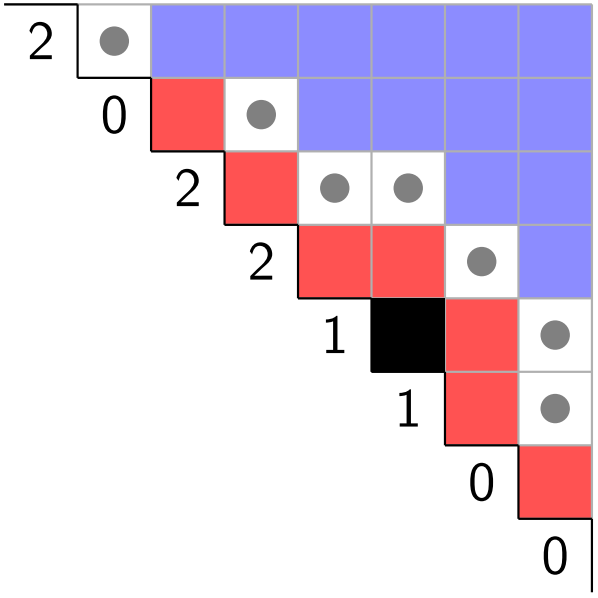
$R_+ \setminus R_q$

$R_q \setminus R_t$

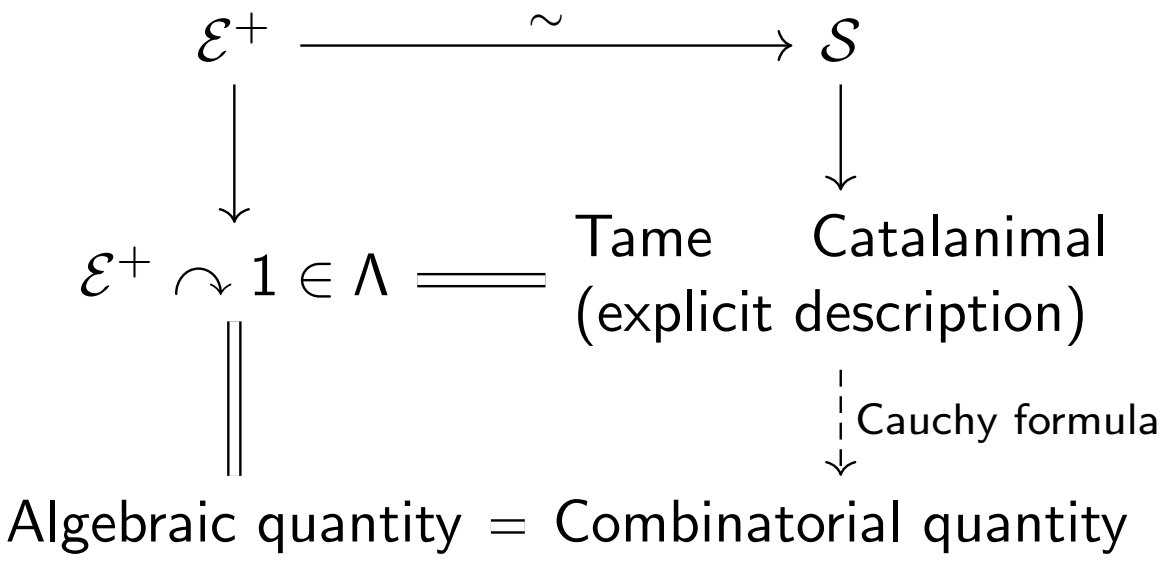
$R_t \setminus R_{qt}$

R_{qt}





Catalanimals and shuffle theorems



Mystery